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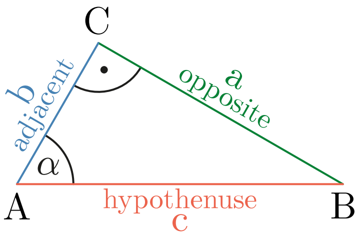
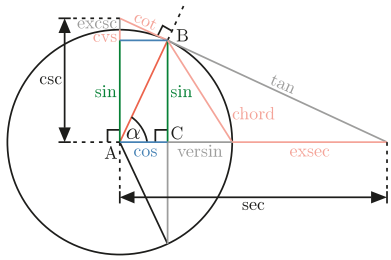
Memorandum № 3

Trigonometric Functions

Note: All angles in this paper are given in radians, unless otherwise stated.

1 Trigonometric functions and their definitions





(Image Credit: [3])

Function	Abbreviation	Geometric Description	Identities
Sine	$\sin(\alpha)$	$\sin\alpha = \frac{\text{opposite}}{\text{hypothenuse}} = \frac{a}{c}$	$\sin\alpha = \cos\left(\alpha - \frac{\pi}{2}\right)$
Cosine	$\cos(\alpha)$	$\cos\alpha = \frac{\text{adjacent}}{\text{hypothenuse}} = \frac{b}{c}$	$\cos\alpha = \sin\left(\alpha + \frac{\pi}{2}\right)$
Tangent	$\tan(\alpha)$	$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \cot\left(\frac{\pi}{2} - \alpha\right) = \frac{1}{\cot\alpha}$
Cotangent	$\cot(\alpha)$	$\cot\alpha = \frac{\text{adjacent}}{\text{opposite}} = \frac{b}{a}$	$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \tan\left(\frac{\pi}{2} - \alpha\right) = \frac{1}{\tan\alpha}$
Secant	$\sec(\alpha)$	$\sec\alpha = \frac{\text{hypothenuse}}{\text{adjacent}} = \frac{c}{b}$	$\sec\alpha = \csc\left(\frac{\pi}{2} - \alpha\right) = \frac{1}{\cos\alpha}$
Cosecant	$\csc(\alpha)$	$\csc\alpha = \frac{\text{hypothenuse}}{\text{opposite}} = \frac{c}{a}$	$\csc\alpha = \sec\left(\frac{\pi}{2} - \alpha\right) = \frac{1}{\sin\alpha}$

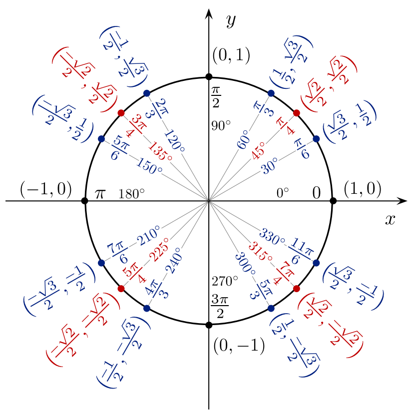
2 Shorthands

Name	Shorthand
versed sine (*versine*)	$\operatorname{versin}\alpha = 2\sin^2\frac{\alpha}{2} = 1 - \cos\alpha$
versed cosine (*vercosine*)	$\operatorname{vercosin}\alpha = 2\cos^2\frac{\alpha}{2} = 1 + \cos\alpha$
coversed sine (*coversine*)	$\operatorname{coversin}\alpha = 1 - \sin\alpha$
coversed cosine (*covercosine*)	$\operatorname{covercosin}\alpha = 1 + \sin\alpha$
half versed sine (*haversine*)	$\operatorname{haversin}\alpha = \frac{1-\cos\alpha}{2}$
half versed cosine (*havercosine*)	$\operatorname{havercosin}\alpha = \frac{1-\sin\alpha}{2}$

Name	Shorthand
half coversed sine (*hacoversine*)	$\operatorname{hacoversin}\alpha = \frac{1-\sin\alpha}{2}$
half coversed cosine (*hacovercosine*)	$\operatorname{hacovercosin}\alpha = \frac{1+\sin\alpha}{2}$
exterior secant (*exsecant*)	$\operatorname{exsec}\alpha = \sec\alpha - 1$
exterior cosecant (*excosecant*)	$\operatorname{excsc}\alpha = \csc\alpha - 1$
chord	$\operatorname{crd}\alpha = 2\sin\frac{\alpha}{2}$
sinus cardinalis (*cardinal sine*)	$\operatorname{sinc}\alpha = \frac{\sin(\pi\alpha)}{\pi\alpha}$ (normalized)
	$\operatorname{sinc}\alpha = \frac{\sin\alpha}{\alpha}$ (unnormalized)

3 Important values of the basic trigonometric functions

	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$	2π
	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\cot\alpha$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\pm\infty$



(Image Credit: [4])

Mnemonics for sine and cosine:

	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$	2π
	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
$\sin\alpha$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$	$-\frac{\sqrt{1}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{4}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
$\cos\alpha$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$	$-\frac{\sqrt{1}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{4}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$

Errors, comments or ideas regarding this paper?
→ https://goo.gl/XB85LZ

4 Pythagorean and related trigonometric identities

$\cos^2\alpha + \sin^2\alpha = 1$

(Pythagorean Identity, named after PYTHAGORAS of Samos Ὁ Πυθαγόρας ὁ Σάμιος)

$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \sec^2\alpha$$

$$1 + \cot^2\alpha = \frac{1}{\sin^2\alpha} = \csc^2\alpha$$

in terms of	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$	$\sec\alpha$	$\csc\alpha$
$\sin\alpha =$	$\sin\alpha$	$\pm\sqrt{1-\cos^2\alpha}$	$\pm\frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}$	$\pm\frac{1}{\sqrt{1+\cot^2\alpha}}$	$\pm\frac{\sqrt{\sec^2\alpha-1}}{\sec\alpha}$	$\frac{1}{\csc\alpha}$
$\cos\alpha =$	$\pm\sqrt{1-\sin^2\alpha}$	$\cos\alpha$	$\pm\frac{1}{\sqrt{1+\tan^2\alpha}}$	$\pm\frac{\cot\alpha}{\sqrt{1+\cot^2\alpha}}$	$\frac{1}{\sec\alpha}$	$\pm\frac{\sqrt{\csc^2\alpha-1}}{\csc\alpha}$
$\tan\alpha =$	$\pm\frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}$	$\pm\frac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}$	$\tan\alpha$	$\frac{1}{\cot\alpha}$	$\pm\sqrt{\sec^2\alpha-1}$	$\pm\frac{1}{\sqrt{\csc^2\alpha-1}}$
$\cot\alpha =$	$\pm\frac{\sqrt{1-\sin^2\alpha}}{\sin\alpha}$	$\pm\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$	$\frac{1}{\tan\alpha}$	$\cot\alpha$	$\pm\frac{1}{\sqrt{\sec^2\alpha-1}}$	$\pm\sqrt{\csc^2\alpha-1}$
$\sec\alpha =$	$\pm\frac{1}{\sqrt{1-\sin^2\alpha}}$	$\frac{1}{\cos\alpha}$	$\pm\sqrt{1+\tan^2\alpha}$	$\pm\frac{\sqrt{1+\cot^2\alpha}}{\cot\alpha}$	$\sec\alpha$	$\pm\frac{\csc\alpha}{\sqrt{\csc^2\alpha-1}}$
$\csc\alpha =$	$\frac{1}{\sin\alpha}$	$\pm\frac{1}{\sqrt{1-\cos^2\alpha}}$	$\pm\frac{\sqrt{1+\tan^2\alpha}}{\tan\alpha}$	$\pm\sqrt{1+\cot^2\alpha}$	$\pm\frac{\sec\alpha}{\sqrt{\sec^2\alpha-1}}$	$\csc\alpha$

5 Symmetry, shifts and periodicity

Symmetry

$\sin(-\alpha) = -\sin\alpha$

$\cos(-\alpha) = \cos\alpha$

$\tan(-\alpha) = -\tan\alpha$

$\cot(-\alpha) = -\cot\alpha$

$\sec(-\alpha) = \sec\alpha$

$\csc(-\alpha) = -\csc\alpha$

Shifts

$x =$	$\frac{\pi}{2}-\alpha$	$\pi-\alpha$	$\alpha+\frac{\pi}{2}$	$\alpha+\pi$	$\alpha+2\pi$
$\sin x =$	$\cos\alpha$	$\sin\alpha$	$\cos\alpha$	$-\sin\alpha$	$\sin\alpha$
$\cos x =$	$\sin\alpha$	$-\cos\alpha$	$-\sin\alpha$	$-\cos\alpha$	$\cos\alpha$
$\tan x =$	$\cot\alpha$	$-\tan\alpha$	$-\cot\alpha$	$\tan\alpha$	$\tan\alpha$
$\cot x =$	$\tan\alpha$	$-\cot\alpha$	$-\tan\alpha$	$\cot\alpha$	$\cot\alpha$
$\sec x =$	$\csc\alpha$	$-\sec\alpha$	$-\csc\alpha$	$-\sec\alpha$	$\sec\alpha$
$\csc x =$	$\sec\alpha$	$\csc\alpha$	$\sec\alpha$	$-\csc\alpha$	$\csc\alpha$

Periodicity
($k \in \mathbb{Z}$)

$\sin(\alpha + k\cdot 2\pi) = \sin\alpha$

$\cos(\alpha + k\cdot 2\pi) = \cos\alpha$

$\tan(\alpha + k\cdot \pi) = \tan\alpha$

$\cot(\alpha + k\cdot \pi) = \cot\alpha$

$\sec(\alpha + k\cdot 2\pi) = \sec\alpha$

$\csc(\alpha + k\cdot 2\pi) = \csc\alpha$

6 Addition theorems, sum and difference identities of two functions

$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha\pm\beta) = \frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$

$\cot(\alpha\pm\beta) = \frac{\cot\alpha\cot\beta\mp1}{\cot\beta\pm\cot\alpha}$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha - \sin\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\tan\alpha \pm \tan\beta = \frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta}$

$\cot\alpha \pm \cot\beta = \pm\frac{\sin(\alpha\pm\beta)}{\sin\alpha\sin\beta}$

$\tan\alpha + \cot\beta = \frac{\cos(\alpha-\beta)}{\cos\alpha\sin\beta}$

$\cot\alpha - \tan\beta = \frac{\cos(\alpha+\beta)}{\sin\alpha\cos\beta}$

7 Products of two functions

$\sin\alpha\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$

$\cos\alpha\sin\beta = \frac{1}{2}\left(\sin(\alpha-\beta) - \sin(\alpha+\beta)\right)$

$\sin\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha-\beta) + \sin(\alpha+\beta)\right)$

$\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)$

8 Power reduction formula

	Sine	Cosine
n odd	$\sin^n\alpha = \frac{2}{2^n}\sum_{k=0}^{\frac{n-1}{2}}(-1)^{\left(\frac{n-1}{2}-k\right)}\binom{n}{k}\sin\left((n-2k)\alpha\right)$	$\cos^n\alpha = \frac{2}{2^n}\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}\cos\left((n-2k)\alpha\right)$
n even	$\sin^n\alpha = \frac{1}{2^n}\binom{n}{\frac{n}{2}} + \frac{2}{2^n}\sum_{k=0}^{\frac{n}{2}-1}(-1)^{\left(\frac{n}{2}-k\right)}\binom{n}{k}\cos\left((n-2k)\alpha\right)$	$\cos^n\alpha = \frac{1}{2^n}\binom{n}{\frac{n}{2}} + \frac{2}{2^n}\sum_{k=0}^{\frac{n}{2}-1}\binom{n}{k}\cos\left((n-2k)\alpha\right)$

$$\sin^2\alpha = \frac{1-\cos(2\alpha)}{2} \qquad \sin^3\alpha = \frac{3\sin\alpha - \sin(3\alpha)}{4} \qquad \sin^4\alpha = \frac{\cos(4\alpha) - 4\cos(2\alpha) + 3}{8}$$

$$\cos^2\alpha = \frac{1+\cos(2\alpha)}{2} \qquad \cos^3\alpha = \frac{\cos(3\alpha) + 3\cos\alpha}{4} \qquad \cos^4\alpha = \frac{\cos(4\alpha) + 4\cos(2\alpha) + 3}{8}$$

9 Multiple-angle formulæ

$$\sin(n\theta) = \sum_{k=0}^{n} \binom{n}{k} \cos^k \theta \sin^{n-k} \theta \sin \frac{\pi(n-k)}{2} \qquad \tan((n+1)\theta) = \frac{\tan(n\theta)+\tan\theta}{1-\tan(n\theta)\tan\theta}$$

$$\cos(n\theta) = \sum_{k=0}^{n} \binom{n}{k} \cos^k \theta \sin^{n-k} \theta \cos \frac{\pi(n-k)}{2} \qquad \cot((n+1)\theta) = \frac{\cot(n\theta)\cot\theta-1}{\cot(n\theta)+\cot\theta}$$

	$\sin x$	$\cos x$	$\tan x$	$\cot x$
$x = 2\alpha$	$2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$	$\cos^2\alpha - \sin^2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$	$\frac{2\tan\alpha}{1-\tan^2\alpha}$	$\frac{\cot^2\alpha-1}{2\cot\alpha}$
$x = 3\alpha$	$3\sin\alpha - 4\sin^3\alpha$	$4\cos^3\alpha - 3\cos\alpha$	$\frac{3\tan\alpha-\tan^3\alpha}{1-3\tan^2\alpha}$	$\frac{\cot^3\alpha-3\cot\alpha}{3\cot^2\alpha-1}$
$x = 4\alpha$	$8\cos^3\alpha\sin\alpha - 4\cos\alpha\sin\alpha$	$8\cos^4\alpha - 8\cos^2\alpha + 1$	$\frac{4\tan\alpha-4\tan^3\alpha}{1-6\tan^2\alpha+\tan^4\alpha}$	$\frac{\cot^4\alpha-6\cot^2\alpha+1}{4\cot^3\alpha-4\cot\alpha}$

10 Half-angle formulæ

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1}{2}(1-\cos\alpha)} \qquad \tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha} = \csc\alpha - \cot\alpha$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1}{2}(1+\cos\alpha)} \qquad \cot\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \frac{1+\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1-\cos\alpha} = \csc\alpha + \cot\alpha$$

11 Inverse trigonometric functions

11.1 Definition of the inverse trigonometric functions

Trig. function	Name of inverse fct.	Abrreviation	Domain	Codomain
sine	**arcsine**	$\arcsin\alpha$	$-1 \leq \alpha \leq 1$	$-\frac{\pi}{2} \leq \arcsin\alpha \leq \frac{\pi}{2}$
cosine	**arccosine**	$\arccos\alpha$	$-1 \leq \alpha \leq 1$	$0 \leq \arccos\alpha \leq \pi$
tangent	**arctangent**	$\arctan\alpha$	$\alpha \in \mathbb{R}$	$-\frac{\pi}{2} < \arctan\alpha < \frac{\pi}{2}$
cotangent	**arccotangent**	$\operatorname{arccot}\alpha$	$\alpha \in \mathbb{R}$	$0 < \operatorname{arccot}\alpha < \pi$
secant	**arcsecant**	$\operatorname{arcsec}\alpha$	$\alpha \leq -1$ or $1 \leq \alpha$	$0 \leq \operatorname{arcsec}\alpha < \frac{\pi}{2}$ or $\frac{\pi}{2} < \operatorname{arcsec}\alpha \leq \pi$
cosecant	**arccosecant**	$\operatorname{arccsc}\alpha$	$\alpha \leq -1$ or $1 \leq \alpha$	$-\frac{\pi}{2} \leq \operatorname{arccsc}\alpha < 0$ or $0 < \operatorname{arccsc}\alpha \leq \frac{\pi}{2}$

11.2 Relationships between trigonometric functions and inverse trigonometric functions

$$\sin(\arcsin\alpha) = \alpha,\ -1 \leq \alpha \leq 1 \qquad \cos(\arccos\alpha) = \alpha,\ -1 \leq \alpha \leq 1$$

$$\arcsin(\sin\alpha) = \begin{cases} \alpha & \text{for } -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2} \\ \pi - \alpha & \text{for } \frac{\pi}{2} \leq \alpha \leq \frac{3\pi}{2} \\ 2\pi\text{-periodic} & \text{cont'd} \end{cases} \qquad \arccos(\cos\alpha) = \begin{cases} \alpha & \text{for } 0 \leq \alpha \leq \pi \\ 2\pi - \alpha & \text{for } \pi \leq \alpha \leq 2\pi \\ 2\pi\text{-periodic} & \text{cont'd} \end{cases}$$

$$\sin(\arccos\alpha) = \cos(\arcsin\alpha) = \sqrt{1-\alpha^2} \qquad \sin(\arctan\alpha) = \frac{\alpha}{\sqrt{1+\alpha^2}}$$

$$\cos(\arctan\alpha) = \frac{1}{\sqrt{1+\alpha^2}} \qquad \tan(\arcsin\alpha) = \frac{\alpha}{\sqrt{1-\alpha^2}}$$

$$\tan(\arccos\alpha) = \frac{\sqrt{1-\alpha^2}}{\alpha}$$

11.3 Relationships among the inverse trigonometric functions

$$\arcsin\alpha = \frac{\pi}{2} - \arccos\alpha = \arctan\frac{\alpha}{\sqrt{1-\alpha^2}} \qquad \arccos\alpha = \frac{\pi}{2} - \arcsin\alpha = \operatorname{arccot}\frac{\alpha}{\sqrt{1-\alpha^2}}$$

$$\arctan\alpha = \frac{\pi}{2} - \operatorname{arccot}\alpha = \arcsin\frac{\alpha}{\sqrt{1+\alpha^2}} \qquad \operatorname{arccot}\alpha = \frac{\pi}{2} - \arctan\alpha = \arccos\frac{\alpha}{\sqrt{1+\alpha^2}}$$

$$\operatorname{arccsc}\alpha = \frac{\pi}{2} - \operatorname{arcsec}\alpha \qquad \arctan\alpha + \arctan\frac{\alpha}{2} = \pm\frac{\pi}{2},\ \alpha \gtrless 0$$

$$\arcsin(-\alpha) = -\arcsin\alpha \qquad \arccos(-\alpha) = \pi - \arccos\alpha$$

$$\arctan(-\alpha) = -\arctan\alpha \qquad \operatorname{arccot}(-\alpha) = \pi - \operatorname{arccot}\alpha$$

$$\operatorname{arcsec}(-\alpha) = \pi - \operatorname{arcsec}\alpha \qquad \operatorname{arccsc}(-\alpha) = -\operatorname{arccsc}\alpha$$

$$\arcsin\frac{1}{\alpha} = \operatorname{arccsc}\alpha \qquad \arccos\frac{1}{\alpha} = \operatorname{arcsec}\alpha$$

$$\arctan\frac{1}{\alpha} = \begin{cases} \operatorname{arccot}\alpha & \text{for } \alpha > 0 \\ -\pi + \operatorname{arccot}\alpha & \text{for } \alpha < 0 \end{cases} \qquad \operatorname{arccot}\frac{1}{\alpha} = \begin{cases} \arctan\alpha & \text{for } \alpha > 0 \\ \pi + \arctan\alpha & \text{for } \alpha < 0 \end{cases}$$

$$\operatorname{arcsec}\frac{1}{\alpha} = \arccos\alpha \qquad \operatorname{arccsc}\frac{1}{\alpha} = \arcsin\alpha$$

12 Power series expansions, infinite product formulæ

$$\sin\alpha = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \alpha^{2n+1} = \alpha - \frac{1}{3!}\alpha^3 + \frac{1}{5!}\alpha^5 - \dots \qquad \text{for } x \in \mathbb{R}$$

$$\cos\alpha = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot \alpha^{2n} = 1 - \frac{1}{2!}\alpha^2 + \frac{1}{4!}\alpha^4 - \dots \qquad \text{for } x \in \mathbb{R}$$

$$\sin\alpha = \alpha \prod_{n=1}^{\infty} \left(1 - \frac{\alpha^2}{\pi^2 n^2}\right) \qquad \cos\alpha = \prod_{n=1}^{\infty} \left(1 - \frac{\alpha^2}{\pi^2 (n-\frac{1}{2})^2}\right)$$

13 Relationship to the complex exponential function

$$\operatorname{cis}\alpha = \mathrm{e}^{\mathrm{i}\alpha} = \cos\alpha + \mathrm{i}\sin\alpha$$

(Euler's Formula)

$$\mathrm{e}^{-\mathrm{i}\alpha} = \cos(-\alpha) + \mathrm{i}\sin(-\alpha) = \cos\alpha - \mathrm{i}\sin\alpha$$

$$\mathrm{e}^{\mathrm{i}\pi} = -1$$

(Euler's Identity)

$$\sin\alpha = \frac{\mathrm{e}^{\mathrm{i}\alpha} - \mathrm{e}^{-\mathrm{i}\alpha}}{2\mathrm{i}} \qquad \cos\alpha = \frac{\mathrm{e}^{\mathrm{i}\alpha} + \mathrm{e}^{-\mathrm{i}\alpha}}{2} \qquad \tan\alpha = \frac{\mathrm{e}^{\mathrm{i}\alpha} - \mathrm{e}^{-\mathrm{i}\alpha}}{\mathrm{i}\left(\mathrm{e}^{\mathrm{i}\alpha} + \mathrm{e}^{-\mathrm{i}\alpha}\right)}$$

$$\cot\alpha = \frac{\mathrm{i}\left(\mathrm{e}^{\mathrm{i}\alpha} + \mathrm{e}^{-\mathrm{i}\alpha}\right)}{\mathrm{e}^{\mathrm{i}\alpha} - \mathrm{e}^{-\mathrm{i}\alpha}} \qquad \sec\alpha = \frac{2}{\mathrm{e}^{\mathrm{i}\alpha} + \mathrm{e}^{-\mathrm{i}\alpha}} \qquad \csc\alpha = \frac{2\mathrm{i}}{\mathrm{e}^{\mathrm{i}\alpha} - \mathrm{e}^{-\mathrm{i}\alpha}}$$

$$\arcsin\alpha = -\mathrm{i}\ln\left(\mathrm{i}\alpha + \sqrt{1-\alpha^2}\right)$$

$$\arccos\alpha = -\mathrm{i}\ln\left(\alpha + \sqrt{\alpha^2 - 1}\right)$$

$$\arctan\alpha = \frac{\mathrm{i}}{2}\ln\left(\frac{\mathrm{i}+\alpha}{\mathrm{i}-\alpha}\right)$$

$$\operatorname{arccot}\alpha = \frac{\mathrm{i}}{2}\ln\left(\frac{\alpha-\mathrm{i}}{\alpha+\mathrm{i}}\right)$$

$$\operatorname{arcsec}\alpha = -\mathrm{i}\ln\left(\frac{1}{\alpha} + \sqrt{1 - \frac{\mathrm{i}}{\alpha^2}}\right)$$

$$\operatorname{arccsc}\alpha = -\mathrm{i}\ln\left(\frac{\mathrm{i}}{\alpha} + \sqrt{1 - \frac{1}{\alpha^2}}\right)$$

References

[1] Abramowitz, Milton; Stegun, Irene A.: *Handbook of Mathematical Functions*. 10th printing. U.S. Department of Commerce, National Bureau of Standards, Washington, D.C., USA, December 1972.

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